East China Normal University

STAT 23 Real Analysis

Instructor: Wanzhong Lu

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Home University: Shanghai University of Finance Economics

Semester: June27 to July15, 2022

Course Hour: Monday through Friday, 160 mins per teaching day;

Total Contact Hours: 64 contact hours

Credits: 4

Designated Textbook with ISBN:

Original edition, Thomas, Brian S.; Bruckner, Judith B.; Bruckner, Andrew M., Elementary Real Analysis (First Edition), Pearson Education, Inc., Copyright, 2001,

China Adapted edition, Pearson Education Asia Ltd. And Higher Education Press, Copyright, 2005, ISBN 7-04-017788-9

Course Prerequisite:

At least one semester of calculus is required; two or three semesters are strongly recommended.

**Notes: The course might be moved to online delivery due to COVID-19 pandemic. Students will be notified once such decision is made.*

Course Overview

Beginning now, a series of courses were taught at East China Normal University whose purpose was to present, in an integrated manner, the core areas of analysis. The objective was to make plain the organic unity that exists between the various parts of the subject, and to illustrate the wide applicability of ideas of analysis to other fields of mathematics and science. The text book is written in a manner sufficiently flexible to make it possible to use the book for courses of various lengths and a variety of levels of mathematical sophistication. The purposes are multifold:

1. To present familiar concepts from calculus at a more rigorous level.
2. To introduce concepts that are not studied in elementary calculus but that are needed in more advanced undergraduate courses. This would include such topics as point set theory, uniform continuity of functions, uniform convergence of sequence of functions.
3. To provide students with a level of mathematical sophistication that will prepare them for graduate work in mathematical analysis, or for graduate work in several applied fields such as engineering or economics.
4. To develop many of the topics that the authors feel all students of mathematics should know.

The course will cover Chapters 1-12 in China Adapted edition, Copyright, 2005, Pearson Education Asia Ltd. And Higher Education Press. — 1st ed. of the original edition, Thomas, Brian S.; Bruckner, Judith B.; Bruckner, Andrew M., Elementary Real Analysis (First Edition), Pearson Education, Inc., Copyright, 2001

The course will concentrate on elementary real analysis theory and methods and cover the following topics:

(i)Properties of the real numbers; Sequences; Sets of real numbers;

(ii) Continuous functions; Differentiation; The integral;

(iii) Infinite sums; Sequences and series of functions; Power series;

(iv)The Euclidian Spaces R^n ; Differentiation on R^n; Metric spaces;

Learning Outcomes

Upon completion of this course, students should be able to:

1. Learn to understand the main features of elementary real analysis theory and methods.
2. Learn how to analyze mathematical problems properly.
3. To understand the role of elementary real analysis theory and methods.
4. Gain an understanding of elementary real analysis theory and methods relevant to upper division interdisciplinary courses.
5. To develop the skills necessary to diagnose and analyze real-world problems.
6. To be able to apply their knowledge of techniques presented in this course.
7. To be able to utilize their critical thinking skills to solve challenging real-life problems with careful, logical thought and the appropriate mathematical analysis tools.

Grading Scale and Notes

The following definitions will be used as a guide for the assignment of grades:

Number Grade	Letter Grade	Definitions
94-100	A	Extraordinary distinction, indicating a full mastery of course content and excellent work.
90-93	A-	
87-89	B+	Strong performance demonstrating a high level of attainment, indicating a good comprehension of the course material and the student's full engagement with the course requirements and activities.
84-86	B	
80-83	B-	
77-79	C+	Acceptable performance, demonstrating an adequate and satisfactory comprehension of the course material and the student has met the basic requirements for completing assignments and participating in class activities.
70-76	C	
60-69	D	A marginal performance in the required exercises demonstrating a minimal passing level of attainment.
0-59	F	An unacceptable performance. The F grade indicates that the student's performance has revealed almost no understanding of the course content.

Assessment Policy

Assessment	Final Grade
Quiz	30%
Mid-Term Examination	30%
Final Examination	30%
Attendance	10%

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Course Schedule

Date	Lecture	Reading/Assignments/ Examination
Day 1	Chapter 1 Properties of the real numbers. Introduction, The real number system, Algebraic Structure, Order structure, Bounds, Sups and infs. The Archimedean Property; Inductive Property of N; The rational numbers are Dense; The metric structure of R	P1-22 1.10.5,1.10.10, quiz
Day 2	Chapter 2 Sequences, Introduction; Sequences; Countable sets; Convergence; Divergence; Boundedness Property of limits; Algebra of limits; Order properties of limits; Monotone convergence criterion; Examples of limits; Subsequences; Cauchy convergence criterion	P23-76 2.11.15,2.11.16 quiz
Day 3	Chapter 3 Sets of real numbers. Introduction; Interior points; Isolated points; Points of accumulation; Boundary points; Closed sets; Open sets; Elementary topology; Compactness arguments: Bolzano-Weierstrass property; Cantor's intersection property; Cousin's Property; Compact sets; Countable sets; Dense sets; Nowhere Dense sets	P77-126 3.4.2,3.4.10 quiz
Day 4	Chapter 4 Continuous functions Introduction to limits: limits($\varepsilon-\delta$ definition); limits(sequential definition); limits(mapping definition); One-sided limits; Infinite limits; Properties of limits: Uniqueness of limits; Boundedness of limits; Algebra of limits; Order properties; Composition of functions; Examples; Continuity: How to define continuity; Continuity at a point; Continuity at an arbitrary point; Continuity on a set; Properties of Continuous functions; Uniform continuity; Extremal Properties; Darboux property; Points of discontinuity: Types of discontinuity; Monotonic functions	P127-190 4.4.29,4.9.10 quiz
Day 5	Chapter 5 Differentiation Introduction; The derivative: Definition of the derivative; Differentiability and continuity; The derivative as a magnification; Computations of derivatives: Algebraic rules; The chain rule; Inverse functions; The power rule ; Continuity of the derivative; Local extrema; Mean value theorem: Role's theorem; Mean value theorem; Cauchy's mean value theorem; Monotonicity; The Darboux Property of the derivative; convexity; L'Hopital's rule: L'Hopital's rule:0/0 form; L'Hopital's rule as $x \to \infty$; L'Hopital's rule:∞/∞ form; Taylor Polynomials	P191-250 5.11.4,5.11.8 quiz
Day 6	Chapter 6 The integral Introduction; Cauchy's first method: Scope of Cauchy's first method; Properties of the	P251-288 6.3.2,6.3.3

	integral; Cauchy's second method; Cauchy's second method (continued);The Riemann integral: some examples; Properties of Riemann integral; The improper Riemann integral; More on the fundamental Theorem of calculus	quiz
Day 7	Chapter 7 Infinite sums Part 1: Introduction; Finite sums; Ordered sums: series (Properties & Special series); Criteria for convergence: boundedness criterion； Cauchy Criterion；Absolute convergence; Tests for convergence: trivial test; Direct comparison test; Limit comparison test; Ratio comparison test; d'Alembert's ratio test; Cauchy's root test; Cauchy's condensation test; integral test; Alternating series test	P289-341 7.6.3,7.6.2 quiz
Day 8	Chapter 7 Infinite sums Part 2: Rearrangements: Unconditional convergence; conditional convergence; Products of series: Products of absolutely convergent series; Products of Nonabsolutely convergent series; Infinite Products	P341-370 7.8.1,7.8.4 Mid-Term Examination
Day 9	Chapter 8 Sequences and series of functions Introduction; Pointwise limits; Uniform limits: The Cauchy criterion; Weierstrass M-test; uniform convergence and continuity; Uniform convergence and the integral: Sequences of continuous functions; Sequences of Riemann Integrable functions; Sequence of improper integrals; Uniform convergence and derivatives; Limits of discontinuous derivatives; Pompeiu's function	P371-409 8.3.18,8.3.20 quiz
Day 10	Chapter 9 Power Series Introduction; Power series: convergence; Uniform convergence; Functions represented by power series: continuity of power series; Integration of power series; Derivatives of power series; Power series representations; The Taylor series: representing a function by a Taylor series; Analytic functions; Products of power series; Quotients of power series; Composition of power series; Trigonometric series: Uniform convergence of trigonometric series; Fourier Series; Convergence of Fourier series; Weierstrass approximation theorem	P410-445 9.4.2, 9.4.3 quiz
Day 11	Chapter 10 The Euclidean Spaces R^n The algebraic structure of R^n; The metric structure of R^n; Elementary topology of R^n; Sequences in R^n;	P446-478 10.2.1,10.2.3

	Functions and mappings: Functions from R^n →R; Functions from $R^n \to R^m$; Limits of functions from $R^n \to R^m$; Definition; Coordinate-wise convergence; Algebraic properties; Continuity of functions from $R^n \to R^m$; Compact sets in R^n; Continuous functions on Compact sets	quiz
Day 12	Chapter 11 Differentiation on R^n Part 1: Introduction; Partial and directional derivatives: Partial derivatives; Directional derivatives; Cross partials; Integrals depending on a parameter; Differentiable functions: Approximation by linear functions; Definition of differentiability; Differentiability and continuity; Directional derivatives; An example; Sufficient conditions for differentiability; The differential	P479-510 11.4.4,11.4.1 quiz
Day 13	Chapter 11 Differentiation on R^n Part 2: Chain rules: Preliminary discussion; Informal proof a chain rule; Notation of chain rules; Proofs of chain rules(1); Mean value theorem; Proofs of chain rules(2); Higher derivatives; Implicit functions theorem: One-variable case; several-variable case; Simultaneous Equations; Inverse function theorem; Functions from R → R^m	P510-556 11.5.4(b) quiz
Day 14	Chapter 12 Metric spaces Part 1: Introduction; Metric spaces—specific examples; Additional examples; 12.3.1 Sequence spaces; function spaces; Convergence; Sets in a metric space; Functions: Continuity; Homeomorphisms; Isometrics; Separable space	P557-600 12.1.2 quiz
Day 15	Chapter 12 Metric spaces Part 2: Complete spaces: Completeness proofs; Subsequence of a complete space; Cantor intersection property; Completion of a metric space; Contraction maps; Application of contraction maps (1); Application of contraction maps (2); Systems of Equations; Infinite systems; Integral Equations; Picard's theorem; Compactness: The Bolzano-Weierstrass Property; Continuous functions on compact sets; The Heine-Borel Property; Total boundedness; Compact sets in C[a,b]; Peano's theorem	P600-645 12.8.1 Final Examination

Reading List:

1. Elias M. Stein & Rami Shakarchi, Real analysis: Measure Theory, Integration, & Hilbert Spaces, Copyright 2005, Princeton University Press, Princeton And Oxford, ISBN 978-0-691-11386-9
2. G. B. Folland, Real Analysis: Modern Techniques and Their Applications, 2nd Edition ISBN 978-7-5192-6072-9
3. J. N. McDonald, A course in Real Analysis, 2nd Edition, ISBN 978-7-5100-5263-7